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## Direct and Related Far-Field Inverse Scattering Problems for Spherical Electromagnetic Waves in Chiral Media

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In the Defense of Honesty in Science and Ethics in Engineering*

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# Direct and Related Far-Field Inverse Scattering Problems for Spherical Electromagnetic Waves in Chiral Media

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## Abstract

This paper studies the direct and inverse scattering problem when the incident electromagnetic field is a time harmonic point - generated wave in a chiral media and the scatterer is a perfectly conducting sphere. The exact Green's function and the electric far-field patterns of the scattering problem are constructed. For a small sphere, a closed-form approximation of the scattered wave field at the source of the incident spherical wave is obtained. Also treats the same inverse problem using far-field results via the leading order term in the low-frequency asymptotic expansion of the scattering cross-section.

## Introduction

In a homogeneous isotropic chiral media the electromagnetic fields are composed of left - circularly polarized (LCP) and right - circularly polarized (RCP) components, which have different wave numbers and independent directions of propagation.

The LCP and RCP components are assumed to be spherical Beltrami fields since in practice such wave fields are more readily realized.

In this work, the author has studied the electromagnetic waves in chiral media produced by a point source in the vicinity of the scat-

terer. In particular, in [1], [2], reciprocity, optical and general scattering theorems for stimulation of point-source asymmetric media have been demonstrated. This paper studies the inverse problem of far field [2]. Specifically, we measure the scattering cross-section for a five-point source area.

In the second Section, considering Bohren decomposition into suitable Beltrami fields, we formulate the direct scattering problem of a spherical electromagnetic wave by a perfectly conducting obstacle. This problem is well posed, the existence and

uniqueness has been proved in [1], [3].

In the third Section, after expanding the incident field in terms of spherical wave functions, we obtain the exact solution of the scattering problem as well as an expansion for the electric far-field pattern [2].

Finally in the fourth Section, we consider either LCP or RCP incidence and we obtain an approximation of the scattering cross-section. For the far-field experiments, we measure the scattering cross-section for various point source locations [2].

## Statement of the problem

Our goal is to study the direct and inverse scattering problems when the incident electromagnetic field is a time harmonic point - generated wave in a chiral medium and the scatterer is a perfectly conducting sphere of radius  $a$  centered at the origin. The exterior space ( $r=|\mathbf{r}|>a$ ) is an infinite homogeneous isotropic chiral medium with chirality measure  $\beta$ , electric permittivity  $\epsilon$  and magnetic permeability  $\mu$ .

We consider a time harmonic spherical electromagnetic wave due to a point source at  $P_0$  with position

vector  $\mathbf{r}_0$  with respect to an origin  $O$  in the vicinity of the scatterer. In order to define spherical electromagnetic fields  $\mathbf{E}_{\mathbf{r}_0}$ ,  $\mathbf{H}_{\mathbf{r}_0}$ , we make use of the Bohren decomposition into Beltrami fields  $\mathbf{Q}_{L,\mathbf{r}_0}$  and  $\mathbf{Q}_{R,\mathbf{r}_0}$ , as follows

$$\begin{aligned} \mathbf{E}_{\mathbf{r}_0} &= \mathbf{Q}_{L,\mathbf{r}_0} + \mathbf{Q}_{R,\mathbf{r}_0} \\ \mathbf{H}_{\mathbf{r}_0} &= \frac{1}{i\eta} (\mathbf{Q}_{L,\mathbf{r}_0} - \mathbf{Q}_{R,\mathbf{r}_0}) \end{aligned} \quad (1)$$

where  $\eta = (\mu/\epsilon)^{1/2}$  is the intrinsic impedance of the chiral medium. The Beltrami fields satisfy the equations [4], [5],

$$\begin{aligned} \nabla \times \mathbf{Q}_{L,\mathbf{r}_0} &= \gamma_L \mathbf{Q}_{L,\mathbf{r}_0} \\ \nabla \times \mathbf{Q}_{R,\mathbf{r}_0} &= -\gamma_R \mathbf{Q}_{R,\mathbf{r}_0} \end{aligned} \quad (2)$$

where  $\gamma_L$  and  $\gamma_R$  are wave numbers given by,

$$\gamma_L = \frac{k}{1 - k\beta}, \quad \gamma_R = \frac{k}{1 + k\beta} \quad (3)$$

with  $k = \omega(\epsilon\mu)^{1/2}$ ,  $\omega$  being the angular frequency. The indices L and R denote the LCP and RCP fields respectively. The spherical incident Beltrami fields with suitable normalization have the form [1], [2],

$$\begin{aligned} \mathbf{Q}_{L,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_L) &= \frac{1}{2} \left( \tilde{\mathbf{I}} + \frac{1}{Y_L^2} \nabla \nabla + \frac{1}{Y_L} \nabla \times \tilde{\mathbf{I}} \right) \left( \frac{h_0(Y_L |\mathbf{r} - \mathbf{r}_0|)}{h_0(Y_L r_0)} \right) \cdot \hat{\mathbf{p}}_L = \\ &= \frac{2\pi r_0 e^{-iY_L r_0}}{Y_L} [Y_L \tilde{\mathbf{G}}_{fs}(\mathbf{r}, \mathbf{r}_0) + \nabla \times \tilde{\mathbf{G}}_{fs}(\mathbf{r}, \mathbf{r}_0)] \cdot \hat{\mathbf{p}}_L \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbf{Q}_{R,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_R) &= \frac{1}{2} \left( \tilde{\mathbf{I}} + \frac{1}{Y_R^2} \nabla \nabla - \frac{1}{Y_R} \nabla \times \tilde{\mathbf{I}} \right) \left( \frac{h_0(Y_R |\mathbf{r} - \mathbf{r}_0|)}{h_0(Y_R r_0)} \right) \cdot \hat{\mathbf{p}}_R = \\ &= \frac{2\pi r_0 e^{-iY_R r_0}}{Y_R} [Y_R \tilde{\mathbf{G}}_{fs}(\mathbf{r}, \mathbf{r}_0) - \nabla \times \tilde{\mathbf{G}}_{fs}(\mathbf{r}, \mathbf{r}_0)] \cdot \hat{\mathbf{p}}_R \end{aligned} \quad (5)$$

where  $h_0(x) = h_0^1(x) = e^{ix}/(ix)$  is the zeroth-order spherical Hankel function of first kind,  $\tilde{\mathbf{I}} = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}}$  is the identity dyadic,  $r_0 = |\mathbf{r}_0|$  and  $\tilde{\mathbf{G}}_{fs}(\mathbf{r}, \mathbf{r}_0)$  is the free space dyadic Green function [4]. The constant unit vectors  $\hat{\mathbf{p}}_L$  and  $\hat{\mathbf{p}}_R$  satisfy the relations

$$\begin{aligned} \hat{\mathbf{r}}_0 \cdot \hat{\mathbf{p}}_L &= \hat{\mathbf{r}}_0 \cdot \hat{\mathbf{p}}_R = 0 \\ \hat{\mathbf{r}}_0 \times \hat{\mathbf{p}}_L &= i\hat{\mathbf{p}}_L \\ \hat{\mathbf{r}}_0 \times \hat{\mathbf{p}}_R &= -i\hat{\mathbf{p}}_R \end{aligned} \quad (6)$$

We note that when  $r_0 \rightarrow \infty$ , the incident electric field

$$\begin{aligned} \mathbf{E}_{r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) &= \\ &= \mathbf{Q}_{L,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_L) + \mathbf{Q}_{R,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_R) \end{aligned} \quad (7)$$

reduces to plane electric wave with direction of propagation  $-\hat{\mathbf{r}}_0$  and polarizations  $\hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R$ , since

$$\begin{aligned} \lim_{r_0 \rightarrow \infty} \mathbf{Q}_{L,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_L) &= e^{-iY_L \hat{\mathbf{r}}_0 \cdot \mathbf{r}} \hat{\mathbf{p}}_L = \\ &= \mathbf{Q}_L^{inc}(\mathbf{r}; -\hat{\mathbf{r}}_0, \hat{\mathbf{p}}_L) \end{aligned} \quad (8)$$

$$\begin{aligned} \lim_{r_0 \rightarrow \infty} \mathbf{Q}_{R,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_R) &= e^{-iY_R \hat{\mathbf{r}}_0 \cdot \mathbf{r}} \hat{\mathbf{p}}_R = \\ &= \mathbf{Q}_R^{inc}(\mathbf{r}; -\hat{\mathbf{r}}_0, \hat{\mathbf{p}}_R) \end{aligned} \quad (9)$$

We consider  $\mathbf{E}_{r_0}^{inc}$  is incident upon a perfectly conducting sphere of radius  $a$ . Then, we want to calculate the scattered electric field  $\mathbf{E}_{r_0}^{sc}$ , which is the unique solution of the following exterior boundary value problem

$$\nabla \times \nabla \times \mathbf{E}_{r_0}^{sc}(\mathbf{r}) - 2Y^2 \beta \nabla \times \mathbf{E}_{r_0}^{sc}(\mathbf{r}) - Y^2 \mathbf{E}_{r_0}^{sc}(\mathbf{r}) = \mathbf{0}, \quad r > a \quad (10)$$

$$\hat{\mathbf{n}} \times \mathbf{E}_{r_0}^{sc}(\mathbf{r}) = -\hat{\mathbf{n}} \times \mathbf{E}_{r_0}^{inc}(\mathbf{r}), \quad r = a \quad (11)$$

The Silver-Müller radiation condition is modified as follows

$$\hat{\mathbf{r}} \times \nabla \times \mathbf{E}_{r_0}^{sc}(\mathbf{r}) - \beta Y^2 \hat{\mathbf{r}} \times \mathbf{E}_{r_0}^{sc}(\mathbf{r}) + \frac{iY^2}{k} \mathbf{E}_{r_0}^{sc}(\mathbf{r}) = o\left(\frac{1}{r}\right), \quad r \rightarrow \infty \quad (12)$$

uniformly in all directions  $\hat{\mathbf{r}} \in S^2$ , where  $S^2$  is the unit sphere in  $\mathbb{R}^3$ ,  $\hat{\mathbf{n}}$  is the outward normal unit vector on the scatterer and  $Y^2 = Y_L Y_R$ . The scattered electric field will be depended on the polarizations  $\hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R$  and will have the decomposition

$$\begin{aligned} \mathbf{E}_{r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) &= \mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) \\ &+ \mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) \end{aligned} \quad (13)$$

where  $\mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$  and

$\mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R)$  are the corresponding scattered Beltrami fields which have the following behavior, when  $r \rightarrow \infty$

$$\begin{aligned} \mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) &= h_0(Y_L r) \cdot \\ &\cdot \mathbf{g}_{L,r_0}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) + o\left(\frac{1}{r^2}\right) \end{aligned} \quad (14)$$

$$\begin{aligned} \mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) &= h_0(Y_R r) \cdot \\ &\cdot \mathbf{g}_{R,r_0}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_L, \hat{\mathbf{p}}_R) + o\left(\frac{1}{r^2}\right) \end{aligned} \quad (15)$$

The functions  $\mathbf{g}_{L,r_0}$  and  $\mathbf{g}_{R,r_0}$  are the LCP and RCP far-field patterns respectively [4], [6].

If either a LCP or a RCP spherical electric wave  $\mathbf{E}_{r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_A)$ ,  $A = L, R$ , is incident upon the scatterer, then the scattering cross-section, is given by [6],

$$\begin{aligned} \sigma_{A,r_0}^{sc} &= \int_{S^2} \left[ \frac{1}{Y_L^2} |\mathbf{g}_{L,r_0}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_A)|^2 + \right. \\ &\left. + \frac{1}{Y_R^2} |\mathbf{g}_{R,r_0}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_A)|^2 \right] ds(\hat{\mathbf{r}}) \end{aligned} \quad (16)$$

### Exact Green's function

We take spherical coordinates  $(r, \theta, \varphi)$  where  $\theta \in [0, \pi]$  and  $\varphi \in [0, 2\pi)$ , with the origin at the center of the spherical scatterer, so that the point source is at  $r = r_0, \theta = 0$ .

Thus,  $\mathbf{r}_0 = r_0 \hat{\mathbf{z}}, \hat{\mathbf{p}}_L = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})$

and  $\hat{\mathbf{p}}_R = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})$ , where  $\hat{\mathbf{x}}, \hat{\mathbf{y}}$  and  $\hat{\mathbf{z}}$  are unit vectors in

the  $x, \psi$  and  $z$  directions, respectively. Using spherical vector wave functions, [4], [5], [7], and taking into account (4), and (5), we obtain [2],

$$\mathbf{Q}_{L,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_L) = \sum_{n=1}^{\infty} B_n^L \{ \mathbf{L}_{o1n}^{(1)}(y_L \mathbf{r}) + \mathbf{iL}_{e1n}^{(1)}(y_L \mathbf{r}) \} \quad (17)$$

or

$$\mathbf{Q}_{R,r_0}^{inc}(\mathbf{r} | \hat{\mathbf{p}}_R) = \sum_{n=1}^{\infty} B_n^R \{ \mathbf{R}_{o1n}^{(1)}(y_R \mathbf{r}) - \mathbf{iR}_{e1n}^{(1)}(y_R \mathbf{r}) \} \quad (18)$$

where, for  $r < r_0$

$$B_n^A = \frac{1}{2\sqrt{2}h_0(y_A r_0)} \frac{2n+1}{n(n+1)} H_n(y_A r_0) \quad (19)$$

$$\begin{aligned} \mathbf{E}_{r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L) &= \mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L) + \mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_L) = \\ &= \sum_{n=1}^{\infty} B_n^L a_n^L \{ \mathbf{L}_{o1n}^{(3)}(y_L \mathbf{r}) + \mathbf{iL}_{e1n}^{(3)}(y_L \mathbf{r}) \} + \sum_{n=1}^{\infty} B_n^R a_n^R \{ \mathbf{R}_{o1n}^{(3)}(y_R \mathbf{r}) + \mathbf{iR}_{e1n}^{(3)}(y_R \mathbf{r}) \} \end{aligned} \quad (21)$$

or

$$\begin{aligned} \mathbf{E}_{r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_R) &= \mathbf{Q}_{L,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_R) + \mathbf{Q}_{R,r_0}^{sc}(\mathbf{r} | \hat{\mathbf{p}}_R) + \\ &= \sum_{n=1}^{\infty} B_n^R b_n^R \{ \mathbf{L}_{o1n}^{(3)}(y_L \mathbf{r}) - \mathbf{iL}_{e1n}^{(3)}(y_L \mathbf{r}) \} + \sum_{n=1}^{\infty} B_n^R b_n^R \{ \mathbf{R}_{o1n}^{(3)}(y_R \mathbf{r}) - \mathbf{iR}_{e1n}^{(3)}(y_R \mathbf{r}) \} \end{aligned} \quad (22)$$

Using the boundary condition (11) on  $r = a$ , we obtain [2],

$$H_n(y_A r_0) = h_n(y_A r_0) - \mathbf{i}\tilde{h}_n(y_A r_0)$$

with  $A = L, R$ . The  $h_n$  is a spherical Hankel function of first kind of order  $n$ ,

$$\tilde{h}(x) = x^{-1}h_n(x) + h_n'(x),$$

$\mathbf{L}_{s1n}^{(\rho)}$  and  $\mathbf{R}_{s1n}^{(\rho)}$ , with  $s = e$ , or  $o$  (even or odd) are the spherical functions [4], [5],

$$\mathbf{L}_{s1n}^{(\rho)}(y_L \mathbf{r}) = \mathbf{M}_{s1n}^{(\rho)}(y_L \mathbf{r}) + \mathbf{N}_{s1n}^{(\rho)}(y_L \mathbf{r}) \quad (20)$$

$$\mathbf{R}_{s1n}^{(\rho)}(y_R \mathbf{r}) = \mathbf{M}_{s1n}^{(\rho)}(y_R \mathbf{r}) - \mathbf{N}_{s1n}^{(\rho)}(y_R \mathbf{r})$$

where  $\rho = 1, 3$ , the  $\mathbf{M}_{s1n}^{(\rho)}$  and  $\mathbf{N}_{s1n}^{(\rho)}$  are known spherical vector function [7]. The scattered electric field that comes from a LCP incident field or a RCP incident field has a similar expansion to (17) or to (18)

$$a_n^L = - \frac{j_n(y_L a) \tilde{h}_n(y_R a) + \tilde{j}_n(y_L a) h_n(y_R a)}{h_n(y_L a) \tilde{h}_n(y_R a) + \tilde{h}_n(y_L a) h_n(y_R a)} \quad (23)$$

and

$$a_n^R = - \frac{j_n(y_L a) \tilde{h}_n(y_L a) - \tilde{j}_n(y_L a) h_n(y_L a)}{h_n(y_L a) \tilde{h}_n(y_R a) + \tilde{h}_n(y_L a) h_n(y_R a)} \quad (24)$$

or

$$b_n^L = - \frac{j_n(y_R a) \tilde{h}_n(y_R a) - \tilde{j}_n(y_R a) h_n(y_R a)}{h_n(y_L a) \tilde{h}_n(y_R a) + \tilde{h}_n(y_L a) h_n(y_R a)} \quad (25)$$

and

$$b_n^R = - \frac{j_n(y_R a) \tilde{h}_n(y_L a) + \tilde{j}_n(y_R a) h_n(y_L a)}{h_n(y_L a) \tilde{h}_n(y_R a) + \tilde{h}_n(y_L a) h_n(y_R a)} \quad (26)$$

Using the asymptotic forms [4], [7],

$$\mathbf{L}_{s1n}^{(3)}(y_L \mathbf{r}) \sim \sqrt{n(n+1)} (-i)^n h_0(y_L r) \mathbf{f}_{s1n}^L(\hat{\mathbf{r}}) \quad (27)$$

$$\mathbf{R}_{s1n}^{(3)}(y_R \mathbf{r}) \sim \sqrt{n(n+1)} (-i)^n h_0(y_R r) \mathbf{f}_{s1n}^R(\hat{\mathbf{r}}) \quad (28)$$

where let us introduce LCP Beltrami angular  $\mathbf{f}_{s1n}^L(\hat{\mathbf{r}})$ , and RCP Beltrami angular  $\mathbf{f}_{s1n}^R(\hat{\mathbf{r}})$  [4], satisfy by relations,

$$\mathbf{f}_{s1n}^L(\hat{\mathbf{r}}) = \mathbf{C}_{s1n}(\hat{\mathbf{r}}) + \mathbf{iB}_{s1n}(\hat{\mathbf{r}}), \quad \mathbf{f}_{s1n}^R(\hat{\mathbf{r}}) = \mathbf{C}_{s1n}(\hat{\mathbf{r}}) - \mathbf{iB}_{s1n}(\hat{\mathbf{r}}) \quad (29)$$

we calculate the electric far-field patterns [2],

$$\mathbf{g}_{A,r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_L) = \sum_{n=1}^{\infty} \frac{(2n+1)(-i)^{n-1}}{2\sqrt{2}n(n+1)} \frac{H_n(y_L r_0)}{h_0(y_L r_0)} a_n^A \{ \mathbf{f}_{e1n}^A(\hat{\mathbf{r}}) - \mathbf{i}\mathbf{f}_{o1n}^A(\hat{\mathbf{r}}) \} \quad (30)$$

or

$$\mathbf{g}_{A,r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_R) = \sum_{n=1}^{\infty} \frac{(2n+1)(-i)^{n-1}}{2\sqrt{2}n(n+1)} \frac{H_n(y_R r_0)}{h_0(y_R r_0)} b_n^A \{ -\mathbf{f}_{e1n}^A(\hat{\mathbf{r}}) - \mathbf{i}\mathbf{f}_{o1n}^A(\hat{\mathbf{r}}) \} \quad (31)$$

**A far-field inverse problem**

So far, all of our formulas are exact. In the asymptotic results to follow, there are three parameters  $y_A a$ , with  $A=L, R$  and  $\tau = a/r_0$ . We note that the geometrical parameter  $\tau$  must satisfy  $0 < \tau < 1$  because the point source is outside of the sphere.

We assume that  $|y_A a| \ll 1$ , as well; that is we make the so-called low-frequency assumption. From (23), (24), (26), (27), we obtain [2],

$$\begin{aligned} a_n^L &\sim \frac{1 + \beta k}{2i\zeta_n^2(2n+1)} (y_L a)^{2n+1} \\ a_n^R &\sim -\frac{i}{2n\zeta_n^2} \frac{(1 - \beta k)^{n+2}}{(1 + \beta k)^{n+1}} (y_L a)^{2n+1} \\ y_L a &\rightarrow 0 \end{aligned} \quad (32)$$

or

$$\begin{aligned} b_n^L &\sim -\frac{i}{2n\zeta_n^2} \frac{(1 + \beta k)^{n+2}}{(1 - \beta k)^{n+1}} (y_R a)^{2n+1} \\ b_n^R &\sim -\frac{i(1 - \beta k)}{2\zeta_n^2(2n+1)} (y_R a)^{2n+1} \\ y_R a &\rightarrow 0 \end{aligned} \quad (33)$$

where

$$\zeta_n = 1 \cdot 3 \cdot 5 \cdots (2n-1) = (2n)! / (2^n n!).$$

In particular,

$$\begin{cases} a_1^L = -i \frac{1 + \beta k}{6} (y_L a)^3 + O((y_L a)^5) \\ a_2^L = -i \frac{1 + \beta k}{90} (y_L a)^5 + O((y_L a)^7) \end{cases} \quad y_L a \rightarrow 0 \quad (34)$$

$$\begin{cases} a_1^R = -\frac{i(1 - \beta k)^3}{2(1 + \beta k)^2} (y_L a)^3 + O((y_L a)^5) \\ a_2^R = -\frac{i(1 - \beta k)^4}{36(1 + \beta k)^3} (y_L a)^5 + O((y_L a)^7) \end{cases} \quad y_L a \rightarrow 0 \quad (35)$$

or

$$\begin{cases} b_1^L = -\frac{i(1 + \beta k)^3}{2(1 - \beta k)^2} (y_R a)^3 + O((y_R a)^5) \\ b_2^L = -\frac{i(1 + \beta k)^4}{36(1 - \beta k)^3} (y_R a)^5 + O((y_R a)^7) \end{cases} \quad y_R a \rightarrow 0 \quad (36)$$

$$\begin{cases} b_1^R = -\frac{i(1 - \beta k)}{6} (y_R a)^3 + O((y_R a)^5) \\ b_2^R = -\frac{i(1 - \beta k)}{90} (y_R a)^5 + O((y_R a)^7) \end{cases} \quad y_R a \rightarrow 0 \quad (37)$$

In order to calculate  $\mathbf{g}_{A,r_0}^{sc}$  with an error of  $O((y_A a)^4)$  we only need the following [4]

$$\mathbf{C}_{o11}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{2}} (\cos\varphi \hat{\boldsymbol{\theta}} - \cos\theta \sin\varphi \hat{\boldsymbol{\phi}}) \quad \mathbf{C}_{o12}(\hat{\mathbf{r}}) = (3/2)^{1/2} \cdot (\cos\theta \cos\varphi \hat{\boldsymbol{\theta}} - \cos 2\theta \sin\varphi \hat{\boldsymbol{\phi}}) \quad (40)$$

$$\mathbf{B}_{o11}(\hat{\mathbf{r}}) = -\frac{1}{\sqrt{2}} (\cos\theta \sin\varphi \hat{\boldsymbol{\theta}} + \cos\varphi \hat{\boldsymbol{\phi}}) \quad \mathbf{B}_{e12}(\hat{\mathbf{r}}) = (3/2)^{1/2} \cdot (\cos 2\theta \cos\varphi \hat{\boldsymbol{\theta}} - \cos\theta \sin\varphi \hat{\boldsymbol{\phi}}) \quad (41)$$

$$\mathbf{B}_{e11}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{2}} (\cos\theta \sin\varphi \hat{\boldsymbol{\theta}} - \sin\varphi \hat{\boldsymbol{\phi}}) \quad \mathbf{B}_{o12}(\hat{\mathbf{r}}) = (3/2)^{1/2} \cdot (\cos 2\theta \sin\varphi \hat{\boldsymbol{\theta}} + \cos\theta \cos\varphi \hat{\boldsymbol{\phi}}) \quad (42)$$

$$\mathbf{C}_{e11}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{2}} (-\sin\varphi \hat{\boldsymbol{\theta}} - \cos\theta \cos\varphi \hat{\boldsymbol{\phi}}) \quad \hat{\mathbf{C}}_{e12}(\hat{\mathbf{r}}) = (3/2)^{1/2} \cdot (-\cos\theta \sin\varphi \hat{\boldsymbol{\theta}} - \cos 2\theta \cos\varphi \hat{\boldsymbol{\phi}}) \quad (43)$$

So from (30) and (31), we finally obtain [2],

$$\begin{aligned} \mathbf{g}_{A,r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_A) &= \varpi_A \frac{(1 - \varpi_A \beta k)}{8} [-(y_A a)\tau^2 [\mathbf{f}_{e11}^A(\hat{\mathbf{r}}) + \varpi_A i \mathbf{f}_{o11}^A(\hat{\mathbf{r}})] + \\ &+ (y_A a)^2 \{2i\tau [\mathbf{f}_{e11}^A(\hat{\mathbf{r}}) + \varpi_A i \mathbf{f}_{o11}^A(\hat{\mathbf{r}})] + \frac{i}{\sqrt{3}} \tau^3 [\mathbf{f}_{e12}^A(\hat{\mathbf{r}}) + \varpi_A i \mathbf{f}_{o12}^A(\hat{\mathbf{r}})]\} + \\ &+ (y_A a)^3 \{2[\mathbf{f}_{e11}^A(\hat{\mathbf{r}}) + \varpi_A i \mathbf{f}_{o11}^A(\hat{\mathbf{r}})] + \frac{4}{3\sqrt{3}} \tau^2 [\mathbf{f}_{e12}^A(\hat{\mathbf{r}}) + \varpi_A i \mathbf{f}_{o12}^A(\hat{\mathbf{r}})]\} + \\ &+ O((y_A a)^4) \end{aligned} \quad (44)$$

and

$$\begin{aligned} \mathbf{g}_{A,r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_{A^c}) &= \varpi_A \frac{3(1 - \varpi_A \beta k)^2}{8(1 + \varpi_A \beta k)} (y_A a)\tau^2 [\mathbf{f}_{e11}^A(\hat{\mathbf{r}}) - \varpi_A i \mathbf{f}_{o11}^A(\hat{\mathbf{r}})] + \\ &+ (y_A a)^2 \left\{ -\varpi_A \frac{3i(1 - \varpi_A \beta k)}{4} \tau [\mathbf{f}_{e11}^A(\hat{\mathbf{r}}) - \varpi_A i \mathbf{f}_{o11}^A(\hat{\mathbf{r}})] - \right. \\ &- \varpi_A \frac{5i}{16\sqrt{3}} \frac{(1 - \varpi_A \beta k)^2}{1 + \varpi_A \beta k} \tau^3 [\mathbf{f}_{e12}^A(\hat{\mathbf{r}}) - \varpi_A i \mathbf{f}_{o12}^A(\hat{\mathbf{r}})] \left. \right\} + \\ &+ (y_A a)^3 \left\{ -\varpi_A \frac{3(1 + \varpi_A \beta k)}{4} [\mathbf{f}_{e11}^A(\hat{\mathbf{r}}) - \varpi_A i \mathbf{f}_{o11}^A(\hat{\mathbf{r}})] - \right. \end{aligned} \quad (45)$$

$$- \varpi_A \frac{5(1 - \varpi_A \beta k)}{12\sqrt{3}} \tau^2 [\mathbf{f}_{e12}^A(\hat{\mathbf{r}}) - \varpi_A \mathbf{i} \mathbf{f}_{o12}^A(\hat{\mathbf{r}})] + O((\gamma_A a)^4)$$

where  $\varpi_A = \begin{cases} -1, & A = L \\ 1, & A = R \end{cases}$  and if  $A = L, R$  at that case  $A^c = R, L$ .

Now the scattering cross-section, by LCP or RCP spherical Beltrami fields, from (16), is given by the relations [2],

$$\begin{aligned} \sigma_{L, r_0}^{sc} &= \int_{S^2} \left[ \frac{1}{\gamma_L^2} |\mathbf{g}_{L, r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_L)|^2 + \frac{1}{\gamma_R^2} |\mathbf{g}_{R, r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_L)|^2 \right] ds(\hat{\mathbf{r}}) = \\ &= (\pi a^2) \left\{ \frac{(1 + \beta k)^2}{64} \left[ \frac{16}{3} \tau^4 + (\gamma_L a)^2 \left( \frac{64}{3} \tau^2 + 16\tau^6 \right) + (\gamma_L a)^4 \left( \frac{64}{3} + \frac{16}{9} \tau^4 \right) \right] + \right. \\ &+ \frac{(1 - \beta k)^4}{(1 + \beta k)^2} \left[ \frac{3}{4} \tau^4 + (\gamma_L a)^2 (3\tau^2 + \frac{5(1 - \beta k)^2}{16(1 + \beta k)^2} \tau^6) + \right. \\ &\left. \left. + (\gamma_L a)^4 \left( 3 + \frac{5(1 - \beta k)}{9(1 + \beta k)^2} \tau^4 \right) \right] \right\} + O((\gamma_L a)^6), \quad \gamma_L a \rightarrow 0 \end{aligned} \quad (46)$$

or

$$\begin{aligned} \sigma_{R, r_0}^{sc} &= \int_{S^2} \left[ \frac{1}{\gamma_L^2} |\mathbf{g}_{L, r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_R)|^2 + \frac{1}{\gamma_R^2} |\mathbf{g}_{R, r_0}^{sc}(\hat{\mathbf{r}} | \hat{\mathbf{p}}_R)|^2 \right] ds(\hat{\mathbf{r}}) = \\ &= (\pi a^2) \left\{ \frac{(1 - \beta k)^2}{64} \left[ \frac{16}{3} \tau^4 + (\gamma_R a)^2 \left( \frac{64}{3} \tau^2 + 16\tau^6 \right) + (\gamma_R a)^4 \left( \frac{64}{3} + \frac{16}{9} \tau^4 \right) \right] + \right. \\ &+ \frac{(1 + \beta k)^4}{(1 - \beta k)^2} \left[ \frac{3}{4} \tau^4 + (\gamma_R a)^2 (3\tau^2 + \frac{5(1 + \beta k)^2}{16(1 - \beta k)^2} \tau^6) + \right. \\ &\left. \left. + (\gamma_R a)^4 \left( 3 + \frac{5(1 + \beta k)}{9(1 - \beta k)^2} \tau^4 \right) \right] \right\} + O((\gamma_R a)^6), \quad \gamma_R a \rightarrow 0 \end{aligned} \quad (47)$$

In the special case  $r_0 \rightarrow \infty$  ( $\tau \rightarrow 0$ ), by the relations (46), (47) we obtain [2],

$$\sigma = (\pi a^2) \left\{ \frac{(1 + \beta k)^2}{3} + \frac{3(1 - \beta k)^4}{(1 + \beta k)^2} \right\} (\gamma_L a)^4 + O((\gamma_L a)^6), \quad \gamma_L a \rightarrow 0 \quad (48)$$

or

$$\sigma = (\pi a^2) \left\{ \frac{(1 - \beta k)^2}{3} + \frac{3(1 + \beta k)^4}{(1 - \beta k)^2} \right\} (\gamma_R a)^4 + O((\gamma_R a)^6), \quad \gamma_R a \rightarrow 0 \quad (49)$$

likewise in the case  $\gamma_L a \rightarrow 0$  and  $\gamma_R a \rightarrow 0$ , by the relations (46), (47) we obtain [2],

$$\sigma_{A, r_0}^{sc} = \frac{1}{4} f_A(\beta, k) (\pi a^2) (a/r_0)^4 \quad (50)$$

$$\text{where } A = L, R, \text{ with } f_A(\beta, k) = \frac{(1 - \varpi_A \beta k)^2}{3} + \frac{3(1 + \varpi_A \beta k)^4}{(1 - \varpi_A \beta k)^2} \quad (51)$$

Choose a Cartesian coordinate system  $Ox\psi z$ , and five point-source locations, namely  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  and  $(0, 0, 2l)$ , which are at (unknown) distances  $r_0, r_1, r_2, r_3$  and  $r_4$ , respectively from the sphere's center. The parameter  $l$  is a chosen fixed length. For each location, measure the leading order term in the low-frequency expansion of the scattering cross-section.

Thus, our five measurements are [2],

$$m_j = \frac{1}{4} f_A(\beta, k) \pi a^2 \left( \frac{a}{r_j} \right)^4 \quad (52)$$

$$j = 0, 1, 2, 3, 4$$

Dimensionless quantities related to  $m_j$  are

$$\gamma_j = \frac{1}{\sqrt{m_j}} = 2 \sqrt{\frac{1}{f_A(\beta, k) \pi}} \frac{1}{a} \left( \frac{r_j}{a} \right)^2 \quad (53)$$

$$j = 0, 1, 2, 3, 4$$

equivalently, we obtain [2],

$$r_j^2 = \frac{1}{2} \sqrt{f_A(\beta, k) \pi} \frac{a^3}{l} \gamma_j \quad (54)$$

$$j = 0, 1, 2, 3, 4$$

There are six unknowns namely  $r_0, r_1, r_2, r_3, r_4$  and  $a$ . Furthermore,  $r_0, r_3$  and  $r_4$  are related using the cosine rule,  $r_4^2 + r_0^2 = 2r_3^2 + 2l^2$ . So, we can find the six unknowns. The center of the spherical scatterer is obtained from the intersection of the four spheres centers at  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$ , with corresponding radius  $r_0, r_1, r_2, r_3$  respectively.



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## Preprint Versions

"Direct and inverse scattering problems for spherical Electromagnetic waves in chiral media"  
 Christodoulos Athanasiadis and Nikolaos Berketis  
 "http://arxiv.org/abs/0812.2169"

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#### FRONT COVER VIGNETTE

A faded synthesis of an anthemion rooted in a meandros

The thirteen-leaf is a symbol for a life tree leaf.  
"Herakles and Kerberos", ca. 530–500 BC,  
by Paseas, the Kerberos Painter,  
Museum of Fine Arts, Boston.

[www.mfa.org/collections/object/plate-153852](http://www.mfa.org/collections/object/plate-153852)

The simple meandros is a symbol for eternal immortality.  
"Warrior with a phiale", ca. 480–460 BC,  
by Berliner Maler,  
Museo Archeologico Regionale "Antonio Salinas" di Palermo.

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